Mitigate Overestimation of Voltage Stability Margin by Coupled Single-Port Circuit Models

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***Abstract*—Wide-area measurement-based voltage stability assessment (VSA) by coupled single-port circuit models has been widely discussed recently. This method models the coupling effects of load buses within a meshed network into extra impedance of a single-port model for each load bus. In simulation studies, overestimations of voltage stability margin using this approach have been observed when critical load bus or buses are decoupled from other load buses. In this paper, the overestimations are reported for the first time through examples and are further analyzed in details. Moreover, to mitigate such overestimations, two methods are proposed: one method uses a mitigation factor based on actual system reactive power response; the other method changes the types of certain weak generation buses when forming the coupled impedance. Both approaches are applied to a sample 4-bus system as well as the IEEE 118-bus system and successfully mitigate the overestimations.**

***Index Terms*—Coupled single-port circuit, Measurement-based approach, Phasor measurement units (PMU), Voltage stability, Wide-area measurement system.**

I. INTRODUCTION

Voltage stability is one of the major concerns in modern power systems. It is required that sufficient voltage stability margin needs to be maintained for stable and reliable system operation.

In recent years, enabled by the development of wide-area measurement system (WAMS) consists of phasor measurement units (PMU), measurement-based voltage stability assessment (VSA) and control approaches have been vastly studied [1]–[5]. These relative new approaches, compared with conventional model-based counterparts, are simple and fast which makes them good candidates for real-time applications.

Among various measurement-based VSA approaches, the coupled single-port circuit (CSPC) method is a recently developed technique first proposed in [4]. The original CSPC method is modified in [6] by introducing series compensation of a shunt admittance in order to make the CSPC method applicable to non-proportional load increasing pattern. One big advantage of CSPC method is that it explicitly models the coupling effects of other loads and generator buses through the system admittance/impedance matrix. This feature makes CSPC method more versatile than those model-free measurement-based approaches. Based on CSPC method, a recent work [7] proposes a wide-area measurement-based loading margin sensitivity and extends measurement-based approaches to voltage stability control applications.

As reported in [4], [8], CSPC method achieves decent results in standard as well as practical test cases benchmarked with classic model-based approaches. However, in our simulation studies, overestimations of voltage stability margin under certain conditions are observed. Such overestimation has never been reported in literature before, most likely because that the occurrence is not often. However, if such overestimation occurs, it will provide an over optimistic assessment to the system operators and will fail to detect critical system operation conditions.

In this work, the overestimation is reported and analyzed in details. Moreover, two methods to mitigate such overestimations are provided. The remaining part of this paper is organized as follows. Section II introduces the voltage stability assessment based on CSPC method. In Section III, the overestimation is analyzed in details on a sample 4-bus system and the IEEE 118-bus system. Two methods to mitigate the reported overestimations are provided in Section IV. Section V presents the simulation results. Conclusions are drawn in Section VI.

II. VOLTAGE STABILITY ASSESSMENT USING COUPLED SINGLE-PORT CIRCUIT METHOD

A meshed power system can be modeled as a multi-port network according to Kirchhoff's current law [4]

$$\begin{bmatrix} -\boldsymbol{I}_L \\ 0 \\ \boldsymbol{I}_G \end{bmatrix} = [\boldsymbol{Y}] \begin{bmatrix} \boldsymbol{V}_L \\ \boldsymbol{V}_T \\ \boldsymbol{V}_G \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{LL} & \boldsymbol{Y}_{LT} & \boldsymbol{Y}_{LG} \\ \boldsymbol{Y}_{TL} & \boldsymbol{Y}_{TT} & \boldsymbol{Y}_{TG} \\ \boldsymbol{Y}_{GL} & \boldsymbol{Y}_{GT} & \boldsymbol{Y}_{GG} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_L \\ \boldsymbol{V}_T \\ \boldsymbol{V}_G \end{bmatrix} \quad (1)$$

where the $\boldsymbol{Y}$ matrix is the system admittance matrix, $\boldsymbol{V}$ and $\boldsymbol{I}$ are vectors of voltage and current phasors, and the subscript L, T, and G represent load bus, tie bus (the buses with no current injection to), and generator bus, respectively.

Eliminating the voltage vectors of the tie buses:

$$\begin{aligned} \boldsymbol{V}_L &= \boldsymbol{K}\boldsymbol{V}_G - \boldsymbol{Z}_{LL}\boldsymbol{I}_L \\ \boldsymbol{Z}_{LL} &= (\boldsymbol{Y}_{LL} - \boldsymbol{Y}_{LT}\boldsymbol{Y}_{TT}^{-1}\boldsymbol{Y}_{TL})^{-1} \\ \boldsymbol{K} &= \boldsymbol{Z}_{LL}(\boldsymbol{Y}_{LT}\boldsymbol{Y}_{TT}^{-1}\boldsymbol{Y}_{TG} - \boldsymbol{Y}_{LG}) \end{aligned} \tag{2}$$

For each load bus i:

$$\boldsymbol{V}_{Li} = \boldsymbol{E}_{eq,i} - (\boldsymbol{Z}_{LL,i} + \boldsymbol{Z}_{coupled,i})\boldsymbol{I}_{Li} = \boldsymbol{E}_{eq,i} - \boldsymbol{Z}_{eq,i} \cdot \boldsymbol{I}_{Li} \tag{3}$$

$$\boldsymbol{E}_{eq,i} = \left[\boldsymbol{K}\boldsymbol{V}_G\right]_i, \boldsymbol{Z}_{coupled,i} = \sum_{j=1, j\neq i}^{n} \boldsymbol{Z}_{LLij}\frac{\boldsymbol{I}_{Lj}}{\boldsymbol{I}_{Li}} \tag{4}$$

By modeling the coupling effect of other loads into $Z_{coupled,i}$, the Thevenin equivalent can be found via (3). According to (4), the Thevenin voltage $\boldsymbol{E}_{eqi}$ is determined by the generator terminal voltage with matrix $\boldsymbol{K}$. The Thevenin impedance $\boldsymbol{Z}_{eqi}$ is determined by the diagonal element of $\boldsymbol{Z}_{LL}$ and the coupling of other loads through corresponding off-diagonal elements in $\boldsymbol{Z}_{LL}$.

Assume that the measurements of voltage phasors at all generator terminals and voltage and current phasors at load bus i are available from PMUs, and the system admittance matrix is also available. The Thevenin equivalent seen from bus i can be calculated using (3) and (4).

It is noted in [4] that for the coupled single-port circuit to access the stability margin accurately, the assumption that all the loads are increasing proportionally are made, which keeps the ratio of the current of two load buses nearly constant. Under such assumption, the loading margin of the load bus i can be calculated as follows:

$$\begin{aligned} \lambda_i &= f\left(E_{eq,i}, R_{eq,i}, X_{eq,i}, P_i, Q_i\right) \\ &= \frac{E_{eq,i}{}^2\left(\sqrt{\left(R_{eq,i}^2 + X_{eq,i}^2\right)\left(P_i^2 + Q_i^2\right)} - R_{eq,i}P_i - X_{eq,i}Q_i\right)}{2\left(X_{eq,i}P_i - R_{eq,i}Q_i\right)^2} - 1 \end{aligned} \tag{5}$$

where $R_{eq,i}$ and $X_{eq,i}$ are the real and imaginary parts of $\boldsymbol{Z}_{eqi}$, respectively. P_i and Q_i are, respectively, the real and reactive power consumption at bus i. $E_{eq,i}$ is the magnitude of $\boldsymbol{E}_{eqi}$.

The loading margin of the system is determined by the smallest margin of all load buses

$$\lambda_{sys} = \min\{\lambda_1, \lambda_2, \cdots, \lambda_n\} \tag{6}$$

III. OVERESTIMATIONS OF VOLTAGE STABILITY MARGINS

Although CSPC method achieves decent results in standard as well as practical test systems, overestimation can be observed in certain system topologies under certain load increasing scenarios as explained in this section.

A. *Observed Overestimation in IEEE 118-bus system*

The overestimation is first observed in IEEE 118-bus system [9] when all the loads and generation are increasing proportionally. The loading margins of selected critical buses using CSPC method are shown in Fig. 1. According to Fig. 1, the most critical bus is Bus 44 with the smallest LM of 3.81 which means the system can support 381% of the base load. However, the LM of the system calculated through continuation power flow is 2.19 and is represented by the red line in Fig. 1 (The continuation power flow result used in this study is from MATPOWER 5.1 [10] under MATLAB R2013b environment). It is clear that CSPC method overestimates the voltage stability margin of the IEEE 118-bus system in this case. The mismatch in terms of LM is around 1.62 or 162%.

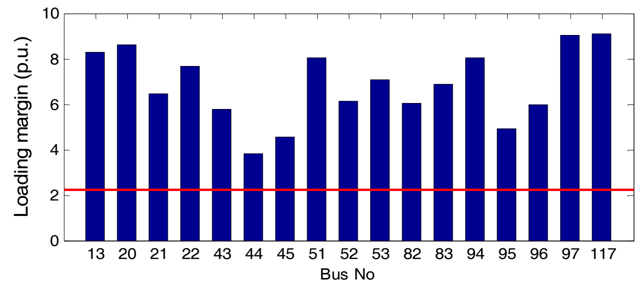

Figure 1. Loading margin of selected buses of IEEE 118-bus system.

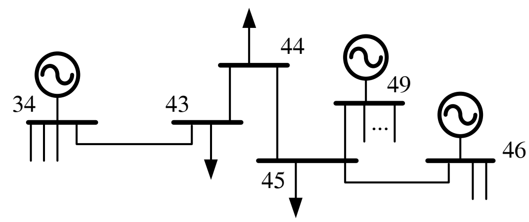

Figure 2. Partial diagram of IEEE 118-bus system near Bus 44.

Further examining the topology of the system around Bus 44, it is found that the small pocket containing Bus 44 is 'isolated' by several generator buses from the outside system as shown in Fig. 2. In Fig. 2, the open lines are branches that connect to the rest of the system. With this special topology, load buses 43, 44, and 45 are decoupled from other load buses because the elements of $\boldsymbol{Z}_{LL}$ in (2) corresponding to the coupling effects of outside load buses are all zero.

B. *Analysis on Sample 4-bus system*

To better demonstrate the correlation between the special isolation topology and the overestimation in voltage stability margin, a sample 4-bus system is built. The topology of the system is shown in Fig. 3. The transmission lines are assumed to be identical with a reactance of 0.2 p.u. Load bus L_1 and L_2 each has a load of (100 MW, 30 MVar) with a 100 MVA base. The scheduled voltage magnitudes at both G_1 and G_2 are set to 1.0 p.u. G_1 (Bus 1) is the slack bus. G_2 (Bus 2) is a PV bus.

In the sample 4-bus system, L_1 is isolated by G_1 and G_2 from the other load bus L_2 and is to mimic the load pocket illustrated in Fig. 2 in the IEEE 118-bus system.

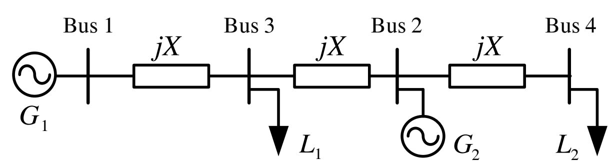

Figure 3. Sample 4-bus system.

Voltage stability studies using both CSPC method and the standard CPF are conducted on this system. Different scenarios of generation dispatch between G_1 and G_2 are considered. Because the resistances are all 0, if G_2 is set to generate PG_2 MW, the real power output of G_1 will be (200 – PG_2) MW. In the simulation, 10 different dispatch scenarios are considered where the output of G_2 increases from 20 MW to 200 MW. All the loads and generations are increased proportionally up until the voltage collapsing point.

The results are shown in Fig. 4. The loading margins (LM) of load bus L_1 and L_2 calculated by the CSPC method are represented by the blue and green bars respectively. The red bars represent the LMs calculated with CPF.

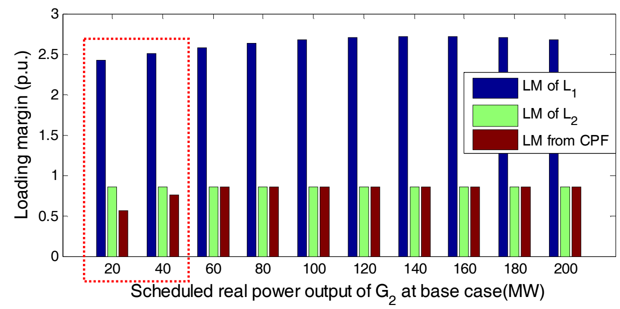

Figure 4. Loading margin of sample 4-bus system.

It is observed that, in the 10 scenarios, according to the CSPC method the critical bus is always L_2 (green bar). When the dispatched real power generation of G_2 at the base case is larger than 60 MW, the results of the CSPC method are consistent with CPF. However, when the output of G_2 at the base case is less than 60 MW, the overestimation is observed where the estimated LMs of both L_1 and L_2 using CSPC are larger than the results from CPF.

Furthermore, for the scenario where the output of G_2 is 20 MW, the load impedances and the estimated Thevenin equivalent (TE) impedances of L_1 and L_2 using CSPC as load increases to up until the voltage collapsing point is plotted in Fig. 5. As load increases, the load impedances (dashed curves) are decreasing while the TE impedances keep constant. Theoretically, at the collapsing point, the TE impedance should equal to the load impedance. However, in Fig. 5, it is clearly that the two impedances of each load bus (same color) do not match at the end. Since the load impedances are the actual values from measurements, it is reasonable to state that the TE impedances are underestimated which eventually causes the overestimation of the voltage stability margin.

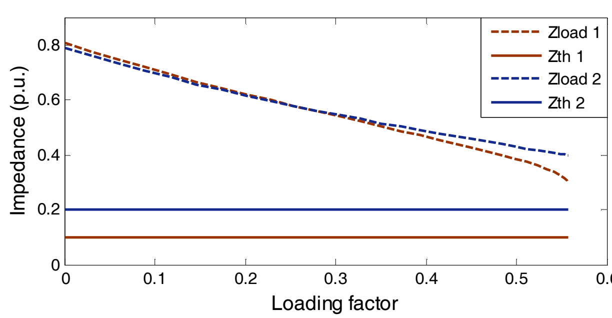

Figure 5. Load impedances v.s. TE impedance as load increases.

The isolation of L_1 to L_2 is just the reason to the underestimation of TE impedance depicted in Fig. 5. According to (4), the coupling impedance of other load buses is related to the corresponding element in $\boldsymbol{Z}_{LL}$. In this system, because of such isolation, the off-diagonal elements of $\boldsymbol{Z}_{LL}$ calculated through (2) are 0 and the TE impedance is just the self-impedance. However, the coupling effects of L_2 to L_1 shall not be completely ignored.

When the output of G_2 is much smaller than the load of L_2, the increase of load at L_2 will not be met by the increase of generation at G_2. Consequently, G_1 will support part of the increased load through Line 1-3 and Line 3-2. This increased flow will take some capacity of the transfer path from G_1 to L_1 and will reduce the voltage stability margin of L_1. By ignoring the coupling effects, the transfer capacity taken by L_2 will not be considered when calculate the voltage stability margin of L_1 which eventually results the illustrated overestimation of stability margin. This is the situation when the real power output of G_2 is 20 and 40 MW.

On the other hand, when G_2 has enough capacity to support L_2, this coupling efforts could be totally ignored since the increase of L_2 will not affect the flow of Line 1-3 or Line 3-2. This explains why as G_2 increases the overestimation disappears.

Moreover, in the critical load pocket of the IEEE 118-bus system depicted in Fig. 2, the generator bus, Bus 34, that isolates the load pocket from the rest of the system has a scheduled real power output of 0. That is to say the adjacent load buses to Bus 34 will easily cast their impacts on the stability margins of Bus 44 bypass Bus 34.

C. *General Comments*

Through the analysis on the 4-bus system and the 118-bus system, it can be concluded that the overestimation tends to happen when the following two conditions are met:

1) The critical load bus or buses are isolated from the rest of load buses by several generator buses as depicted in Fig. 2 or Fig. 3.

2) Among the generator buses that separate the critical load bus or buses, at least one of them is a 'weak' generator bus whose real power output is so small that adjacent load buses will cast their impact bypass this generator bus. In the sample 4-bus system, the weak bus is G_2 and in the IEEE 118-bus system, the weak bus is Bus 34.

When the critical load bus or buses of a system meets the above mentioned two conditions, it is suggested to compare the results of CSPC method with other standard method in order to identify possible overestimation situations.

IV. METHODS TO MITIGATE THE OVERESTIMATIONS

If the aforementioned overestimation is observed, it is crucial that an effective mitigation approach is applied. Otherwise, the overestimation will give the system operators optimistic results and may delay the identification of critical system operating conditions. In this section, two possible approaches to mitigate the overestimations are presented.

A. *Modified Coupled Single-Port Model (MD)*

This approach was originally proposed by Liu and Chu in [6] for compensating underestimation of voltage stability margin using coupled single-port circuit under non-proportional increased load.

In order to compensate the underestimation, a shunt admittance Y_{Ci} is added to the ith equivalent branch as shown in Fig. 6. Essentially, a mitigation factor α_i is multiplied to the TE impedance. α_i can be solved by letting the system reactive power response equal to reactive power response of the equivalent system as shown in Fig. 6.

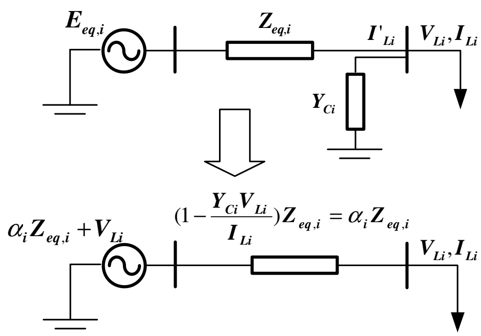

Figure 6. Equivalent series compensation for ith equivalent branch.

The detailed steps for solving α_i used in this work is the same as in [6] and is not elaborated here due to the page limit. Basically, it is to solve a quadratic equation.

A slight modification is made to let the mitigation factor work for the overestimation in this study. In [6], the voltage stability is underestimated which means the TE impedance is overestimated. Therefore, the authors set a feasible range of α_i to (0, 1] which forces the modified TE impedance less than the initial value. While, in order to mitigate the overestimation of voltage stability margin, or the underestimation of TE impedance in this study, the feasible range of α_i should be extended to $(0, \infty)$. When α_i is larger than 1, the mitigation factor mitigates the overestimation introduced in this study, when it is smaller than 1, the mitigation factor compensates the underestimation reported in [6].

B. Negative Load Model(NL)

As explained in Section III, the overestimation of stability margin is caused by isolation of critical load buses by weak generator buses. One straightforward solution is to change type of the weak generator buses to load buses and treat them as negative loads. This way, the isolation of the critical bus or buses is broken and the coupling of outside loads can be modeled into the TE impedance of the critical load since the corresponding elements in $\boldsymbol{Z_{LL}}$ calculated through (2) will be filled by non-zero elements.

Although to change the type of the weak generator buses is very simple, to identify the isolation and the weak generator buses is not an easy task. In this study, we assume this information is known. In the future work, an algorithm to identify the potential isolation and spot the weak generator buses will be explored.

V. SIMULATION RESULTS

In this section, the two methods introduced in Section IV are applied to the sample 4-bus system and the IEEE 118-bus system in order to mitigate the observed overestimations.

A. Application to the Sample 4-bus System

The scenario where the output of G_2 is 20 MW in the base case is considered. As reported in Section III-B, overestimation of voltage stability margin is observed under this scenario. The simulation is conducted from base case to voltage collapsing point using CPF as load proportionally increases.

The impedances of the equivalent circuit for L_1 as load increases are depicted in Fig. 7. The blue dashed curve is the impedance of the load which decreases as load increases. The solid blue curve is the TE impedance solved using original CSPC method. The red curve is the TE impedance solved by Modified Coupled Single-Port Model (MD). The green curve is the TE impedance solved by Negative Load Model (NL).

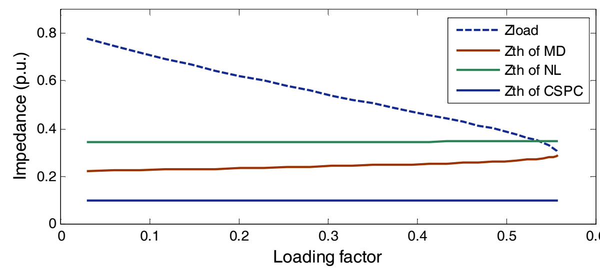

Figure 7. Impedances of equivalent circuit for L1 as load increases.

The TE impedance of the MD method increases as load increases. The mitigation factors α for L_1 is illustrated Fig. 8. The positive solutions are chosen while the negative ones are discarded. As can be seen in Fig. 8, the kept mitigation factors are larger than 2.0 which justifies the necessity of extending the feasible range to $(0, \infty)$. In this case, based on the actual system reactive power response, the MD method successfully identifies the overestimation problem and adjusts the TE impedance in the correct positions. The TE impedance at the final moment meets the load impedance.

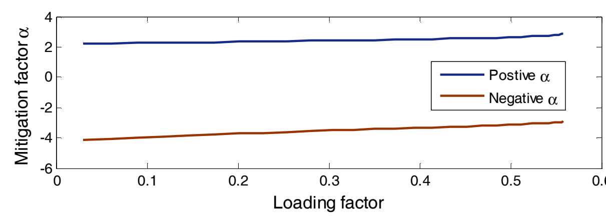

Figure 8. Mitigation factors of L_1 as load increases.

The TE impedance of NL method keeps mostly constant as load increases. This is a more desired feature than MD method in terms of projecting the final collapsing point. However, the estimated TE impedance is slightly larger than the load impedance in the final moment. The impedance matching point actually comes earlier than the actual collapsing point. This means a slightly underestimation of loading margin is brought by NL method.

However, both MD and NL methods improve the performance of the original CSPC method significantly. The loading margins estimated using different methods are demonstrated in Fig. 9. The blue dashed curve is the LM from CPF and serves as the benchmark. The blue solid curve represents the results from original CSPC which is much higher than the correct values. The red curve represents the result from MD and the green curve represents the results from NL. According to Fig. 9 both MD and NL achieve significant improvement in estimating the LM of L_1. MD introduces slightly overestimation while NL introduces slightly underestimation. Moreover, as load increases, the errors tend to reduce.

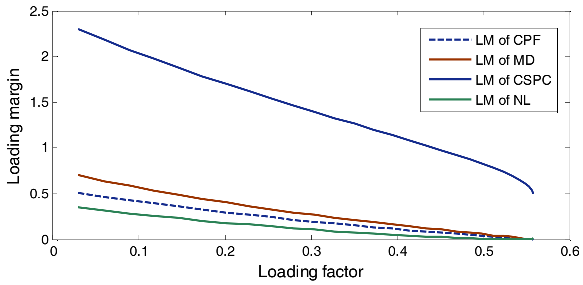

Figure 9. LMs of L_1 by different methods from base case to collapsing point.

B. *Application to IEEE 118-bus System*

Both methods are then applied to the IEEE 118-bus system. When applying NL method, generator buses 34, 36, 41, and 42 are changed to load buses. These 4 generator buses are all generator buses with 0 real power output near critical load buses.

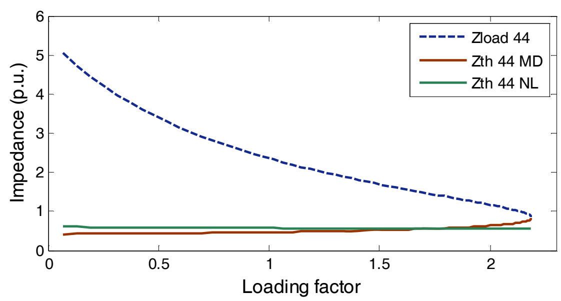

Figure 10. Impedances of equivalent circuit of Bus 44 in 118-bus system

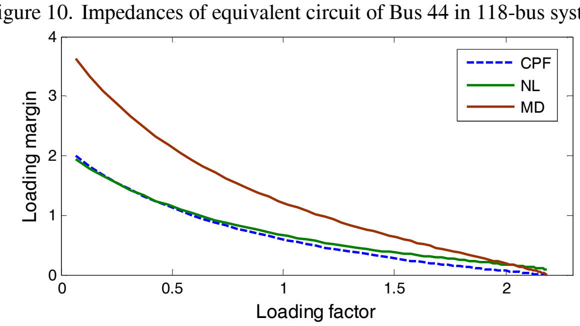

Figure 11. Loading margin of Bus 44 in 118-bus system

The impedances of the equivalent circuit of Bus 44 are shown in Fig. 10. The loading margins using different methods are depicted in Fig. 11. As seen in Fig. 10, the TE impedances are close to the load impedance at the final collapsing moment. According to Fig. 11, NL method estimates the LM more accurately around base case. MD method, though less accurate around base case, is closer to the actual value when approaching the collapsing point. This is because the TE impedance estimated from MD method keeps being updated according to the actual system reactive response. As the operating point approaches closer to the collapsing point, MD approach shall be more accurate. On the other hand, the impedance of NL method keeps almost constant during the whole process. It gives a better estimation around base case, since along the projection, the TE impedance does not change much. However, at the final moment, the impedance of NL method does not exactly match the load impedance.

VI. CONCLUSION

This work reports overestimation of voltage stability margin using coupled single-port circuit (CSPC) method. Through detailed analysis on a sample 4-bus system as well as the IEEE 118-bus system, it can be concluded that the observed overestimation occurs when the critical load bus or buses are isolated from rest of the system by weak generator buses. To mitigate such overestimation, two methods are raised, namely modified coupled single-port model (MD) method and negative load (NL) method. Both methods are applied to the 4-bus and 118-bus systems. Simulation results demonstrate that both methods mitigate the overestimation by original CSPC method. MD method will track the stability margin more accurately near the collapsing point while NL method can give a relative accurate prediction when the operating point is far from collapsing point. The authors suggest that when using CSPC method, it is necessary to check the two conditions summarized in Section III-C to identify potential overestimation problem. If the two conditions are met, the proposed methods should be taken to mitigate the overestimation.

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