System Load Margin Evaluation using Mixed-Integer Conic Optimization

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***Abstract*— This paper proposes a mixed-integer conic optimization approach for estimating system load margin to evaluate static voltage stability in transmission networks. The proposed problem formulation employs the conic quadratic format of power flow equations, and then utilizes a polyhedral approximation to transform the conic quadratic constraints to linear constraints. Also, the complementary constraint of generators' reactive power limit has been considered. The numerical case studies have been performed on IEEE 14, 30, 39 and 118 bus systems, and validated that the proposed method is an effective tool for system load margin evaluation.**

Keywords*— *Static voltage stability, load margin, conic optimization, mixed integer linear programming.

NOMECLATURE

n	Number of system buses.
P_{L0i}, Q_{L0i}	Base active (reactive) power load at bus i.
P_{Gi}, Q_{Gi}	Active (reactive) power generation from conventional generator at bus i.
V_i, θ_i	Voltage magnitude (angle) at bus i.
λ	System load margin.
P_{ij}, Q_{ij}	Active (reactive) power flowing from bus i to j, in p.u.
I_{ij}	Current magnitude flowing from bus i to j, in p.u.
$\boldsymbol{Y}$	Admittance matrix, in p.u.
G_{ij}, B_{ij}	Real (imaginary) part of mutual admittance between bus i and j.
$B_{\text{sh}ij}$	Shunt susceptance in the π-model of line ij.
$P_{Gi}^{\max}$	Maximum active power of generator at bus i.
$P_{Gi}^{\min}$	Minimum active power of generator at bus i.
$V_i^{\max}$	Maximum allowable voltage at bus i.
$V_i^{\min}$	Minimum allowable voltage at bus i.
V_{Gi}^{0}	Reference terminal voltage of generator at bus i.
v_{i1}, v_{i2}	Auxiliary variables representing the changes in the voltage of generator at bus i due to reactive power limit.
$S_{ij}^{\max}$	Maximum apparent power flow from bus i to j, in p.u.
R_{ij}, T_{ij}	Variables associated with line ij in conic model.
u_i	Variables associated with the voltage magnitued of bus i in conic model.
k, v	Parameters of the polyhedral construction.
ψ^k, φ^k	Variables of the polyhedral construction.

I. INTRODUCTION

Recently, due to the deregulation of electricity market, present power systems have to operate close to the limits of thermal and stability more often than ever before. Under this paradigm, as the sustainable increase in electric power demand together with tightening environmental constraints and the competition of electricity market [1, 2], the issue of voltage instability has been more severe and receiving considerable attention. Therefore, in order to relieve or at least minimize the voltage instability issue, providing an efficient and accurate evaluation of system stability margin become necessary.

Generally, the power system load margin is, under a certain operating point, the maximum amount of additional load increases that the system can maintain voltage stability [3]. The load margin is employed as a measure of system security at a given operating point. Also, it can be used as a requirement or constraint in the system reactive power planning [4] or available transfer capability evaluation (ATC) [5, 6]. There are a broad range of previous works on evaluating load margin. The most common method to determine the load margin is continuation power flow (CPF) [7]. The system load level is increased until the power flow cannot converge. However, in a large system, this method may not be efficient and the results are influenced by the choice of step size. And, [8-11] take complementarity constraint of generator reactive power limit into consideration. Thus, the load margin evaluation problem is formulated as an AC optimal power flow (ACOPF) problem with the objective changes to load margin. As shown in [12], the reactive power procurement model is proposed to maintain the voltage stability in which the system load margin is obtained through an optimal power flow (OPF). This particular OPF problem considering voltage stability is always modeled as a mixed integer nonlinear programming (MINLP) problem [13] which brings the challenge to obtain the global optimal solution.

Therefore, in order to better evaluate the system load margin, this paper implements mixed-integer conic optimization approach to transform the original MINLP problem to a mixed integer conic programming (MICP) problem. First, the system power flow equations are reformulated as the second order cone constraints by introducing new variables to represent the relationship of system voltage magnitude and angle in the nodal active and reactive power flow equations [14, 15]. The thermal limits of transmission lines are recast as current limits which can be expressed as linear constraints using the new variables in conic constraints [16, 17]. In addition, the polyhedral approximation is implemented to transform the conic constraints to a set of linear constraints [18]. The complementarity constraints of generators' reactive power limit can be linearized using a set of auxiliary binary variables [8-11]. Moreover, the zero-gap of the conic relaxation has been presented to validate the proposed approach.

The rest of this paper is organized as follows. Section II presents detailed static voltage stability margin evaluation using an optimal power flow model considering the complementarity constraints of generators' reactive power limits. Section III is the conic relaxation model of the above OPF model and the zero-gap proof. Then, the linear model of the conic relaxation is introduced in Section IV. Section V provides case studies and numerical results analysis. Finally, conclusions are outlined in Section VI.

II. Voltage Stability Margin Evaluation Considering Complementarity Constraints

The voltage stability margin is defined as the distance, with respect to the bifurcation parameters λ, from present operating point to the voltage collapse point. If this distance is larger than a desired value [19, 20], the studied system is considered as voltage secure. λ is a scalar bifurcation parameter, typically known as "loading factor". And, as shown in (1) and (2), λ represents the linearly increasing trend in system loading level:

$$P_L = (1+\lambda)\cdot P_{L0} \tag{1}$$

$$Q_L = (1+\lambda)\cdot Q_{L0} \tag{2}$$

where P_{L0} and Q_{L0} are base load power quantities.

The loading factor λ can be computed by reformulating the conventional ACOPF problem with the objective function changed to maximize λ. Hence, the OPF model for evaluating system load margin can be expressed as below:

$$max \quad \lambda \tag{3}$$

s.t.

$$P_{Gi} - (1+\lambda)\cdot P_{L0i} = V_i \sum_{j=1}^{n} V_j (G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)) \tag{4}$$

$$Q_{Gi} - (1+\lambda)\cdot Q_{L0i} = V_i \sum_{j=1}^{n} V_j (G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)) \tag{5}$$

$$P_{ij}^2 + Q_{ij}^2 \le S_{ij}^{\max 2} \tag{6}$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \tag{7}$$

$$0 \le Q_{Gi} - Q_{Gi}^{\min} \perp v_{i1} \ge 0 \tag{8}$$

$$0 \le Q_{Gi}^{\max} - Q_{Gi} \perp v_{i2} \ge 0 \tag{9}$$

$$V_{Gi} = V_{Gi}^0 + v_{i1} - v_{i2} \tag{10}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{11}$$

In this OPF model, (4) and (5) are the nodal active and reactive power flow equations. Constraint (6) is transmission line thermal limits. Constraint (7) limits the generator active power. And constraint (11) maintains all bus voltages within appropriate limits. The complementarity constraints (8) and (9) and the voltage equation (10) represent the effect of reactive power limit on generator voltage. These constraints ensure that all the generators operate at the set terminal voltage as long as the reactive power is within the limits. In this case, the auxiliary variables v_{i1}, v_{i2} will be zero. Meanwhile, If the reactive power output of generator i encounters the minimum limit, v_{i1} will be positive by constraint (8), hence increasing the voltage at this generator bus. Similarly, if the upper limit of reactive power output for generator i is reached, v_{i2} will be positive by constraint (9), therefore reducing the generator terminal voltage according to (10).

This model with its present expression is formulated as a MINLP problem which always brings challenges for reaching the global optimal solution. A lot of research efforts focusing on the transformations to linearize the power flow equations, the quadratic line flow limits and the complementarity constraints of generators' reactive power limits have been performed. Besides, various relaxation methods have also been studied. Thus, in the following, different linearization models for the above nonlinear constraints will be presented, respectively.

First, the complementarity constraints (8) and (9) can be reformulated as a set of linear constraints by adding auxiliary binary variables as shown below:

$$0 \le Q_{Gi} - Q_{Gi}^{\min} \le M_Q^{\min} \gamma_{Gi}^{\min} \tag{12}$$

$$0 \le v_{i1} \le M_Q^{\min}(1-\gamma_{Gi}^{\min}) \tag{13}$$

$$0 \le Q_{Gi}^{\max} - Q_{Gi} \le M_Q^{\max} \gamma_{Gi}^{\max} \tag{14}$$

$$0 \le v_{i2} \le M_Q^{\max}(1-\gamma_{Gi}^{\max}) \tag{15}$$

where $M_Q^{\min}$ and $M_Q^{\max}$ are large enough constants, and $\gamma_{Gi}^{\min}$ and $\gamma_{Gi}^{\min}$ are the auxiliary binary variables [21].

Then, for the line apparent power flow limit, the constraint (6) can be approximated as below, since bus voltage is close to 1 p.u. in transmission system.

$$S_{ij}^2 = [V_i \cdot (I_{ij}^*)]^2 \approx I_{ij}^2 \tag{16}$$

$$I_{ij}^2 = a_{ij}u_i + b_{ij}u_j - 2c_{ij}R_{ij} + 2d_{ij}T_{ij} \le I_{l\max}^2 \tag{17}$$

where a_{ij}, b_{ij}, c_{ij},and d_{ij} are positive line parameters defined in (18)-(21).

$$a_{ij} = G_{ij}^2 + (B_{ij} + B_{\mathrm{sh}ij} / 2)^2 \tag{18}$$

$$b_{ij} = G_{ij}^2 + B_{ij}^2 \tag{19}$$

$$c_{ij} = G_{ij}^2 + B_{ij}(B_{ij} + B_{\mathrm{sh}ij} / 2) \tag{20}$$

$$d_{ij} = G_{ij} B_{\mathrm{sh}ij} / 2 \tag{21}$$

Furthermore, due to the complexity of solving MINLP problem, this model can be simplified to nonlinear programming (NLP) problem through relaxing the complementarity constraints of generators' reactive power limit. The optimistic relaxation (Relaxation 1) is in (22) and (23). And the conservative relaxation (Relaxation 2) is in (22) and (24).

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \tag{22}$$

$$V_i^{\min} \le V_{Gi} \le V_i^{\max} \tag{23}$$

$$V_{Gi} = V_{Gi}^0 \tag{24}$$

The results of these two relaxations will also be presented in the case study. The convex relaxation of power flow equations will be presented in the next section.

III. Mixed-Integer Conic Programming

A. Conic Relaxation of Power Flow Equations

In the above OPF model, the nonlinear and non-convex power flow equations, the line apparent power limit and the complementarity constraints associating the generators' reactive power make the problem hard to solve. In this section, the power flow equation is relaxed using second order cone first, then the proof the zero gap for this conic relaxation is presented after.

The nodal power flow equations are expressed in (25) and (26) below.

$$P_{Gi} - P_{Li} = V_i \sum_{j=1}^{n} V_j (G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)) \tag{25}$$

$$Q_{Gi} - Q_{Li} = V_i \sum_{j=1}^{n} V_j (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)) \tag{26}$$

Define that $u_i = V_i^2$, $T_{ij} = V_i V_j sin\theta_{ij}$, $R_{ij} = V_i V_j cos\theta_{ij}$. Taking the advantage of the specific characteristic of transmission network that V_i is close to 1 p.u. and angle difference between two ends of a branch is very small, it can be epressed that $\theta_{ij} \approx T_{ij}$. So the reformulated power flow equations are below in (27)-(30)

$$P_{Gi} - P_{Li} = -G_{ii} u_i + \sum_{j \in J(i)} (G_{ij} R_{ij} + B_{ij} T_{ij}) \tag{27}$$

$$Q_{Gi} - Q_{Li} = B_{ii} u_i + \sum_{j \in J(i)} (G_{ij} T_{ij} - B_{ij} R_{ij}) \tag{28}$$

$$R_{ij}^2 + T_{ij}^2 = u_i u_j, \boldsymbol{R} \ge 0 \tag{29}$$

$$T_{ij} = \theta_i - \theta_j \tag{30}$$

Till now, this model has been converted as an approximation of the original power flow equations but still not convex yet. While if the equality constraint (29) is relaxed as in (31), the whole power flow equations become a rotating second order cone model which is convex.

$$R_{ij}^2 + T_{ij}^2 \le u_i u_j, \boldsymbol{R} \ge 0 \tag{31}$$

Therefore, the mixed-integer conic programming model (MICP) of maximizing the system load margin can be obtained by the following OPF model. If the complementarity constraints of generators' reactive power limit is relaxed as shown in (22)-(24), the whole model is then a quadratic constrained programming (QCP) problem.

$$max \;\; \lambda \tag{32}$$

$$s.t.$$

$$P_{Gi} - (1+\lambda) \cdot P_{Li} = -G_{ii} u_i + \sum_{j \in J(i)} (G_{ij} R_{ij} + B_{ij} T_{ij}) : \sigma_i \tag{33}$$

$$Q_{Gi} - (1+\lambda) \cdot Q_{Li} = B_{ii} u_i + \sum_{j \in J(i)} (G_{ij} T_{ij} - B_{ij} R_{ij}) : \eta_i \tag{34}$$

$$I_{ij}^2 = a_{ij} u_i + b_{ij} u_j - 2 c_{ij} R_{ij} + 2 d_{ij} T_{ij} \le I_{l\max}^2 : \varsigma_l \tag{35}$$

$$R_l^2 + T_l^2 \le u_i u_j : \mu_l, l \in L \tag{36}$$

$$\theta_l = T_l, l \in L \tag{37}$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \tag{38}$$

$$0 \le Q_{Gi} - Q_{Gi}^{\min} \le M_Q^{\min} \gamma_{Gi}^{\min} \tag{39}$$

$$0 \le u_{i1} \le M_Q^{\min} (1 - \gamma_{Gi}^{\min}) \tag{40}$$

$$0 \le Q_{Gi}^{\max} - Q_{Gi} \le M_Q^{\max} \gamma_{Gi}^{\max} \tag{41}$$

$$0 \le u_{i2} \le M_Q^{\max} (1 - \gamma_{Gi}^{\max}) \tag{42}$$

$$u_{Gi} = u_{Gi}^0 + u_{i1} - u_{i2} \tag{43}$$

$$u_i^{\min} \le u_i \le u_i^{\max} \tag{44}$$

where the variables on the right side of colons are the Lagrange multipliers associating the equality or inequality constraints on the left side of colons. These variables represent the marginal benefit/contribution of each constraint to the system load margin in the above model. σ_i and η_i (dual variables) are the dual of the nodal active and reactive power balance constraints (33) and (34), denoting the sensitivity of the system load margin to a change in active and reactive power demand at bus i; ς_l is the dual of line thermal limit, denoting the sensitivity of the system load margin to a change in the line thermal limit; and μ_l are the dual of the conic constraint, denoting the sensitivity of the system load margin to the conic constraints. The value of μ_l also demonstrate the gap of conic relaxation with the original OPF model. This will be discussed in the next section.

B. Proof of Zero-Gap

It has been proven that the load margin maximization OPF model can obtain zero-gap comparing with the original OPF model in Section II.

As long as the dual variable μ_l is greater than zero, the conic constraints will be binding under this condition and the inequality constraints (31) are resulted as the equality constraints in (29). Consequently the conic relaxation OPF model is zero-gap with the OPF model in (27)-(30). And the calculation error is the approximation error between the power flow equations model in (4)-(5) with the model in (27)-(30). This error is small in a transmission system.

In the following text, the condition of μ_l greater than zero is utilized to prove the zero-gap. The partial derivative of Lagrangian function F_L of the proposed optimization model in (32)-(44) to the variables λ and R_{ij} can be expressed as shown in (45) and (46), respectively.

$$\frac{\partial F_L}{\partial \lambda} = 1 - \sigma_i P_{Li} - \eta_i Q_{Li} = 0 \tag{45}$$

$$\frac{\partial F_L}{\partial R_{ij}} = -G_{ij}\sigma_i + B_{ij}\eta_i + 2c_{ij}\varsigma_{ij} - 2\mu_{ij}R_{ij} = 0 \tag{46}$$

$$\left(\frac{G_{ij}Q_{Li}}{P_{Li}} + B_{ij}\right)\eta_i + \frac{G_{ij}}{P_{Li}} + 2c_{ij}\varsigma_{ij} = 2\mu_{ij}R_{ij} \tag{47}$$

In transmission network, $G_{ij} \approx 0$, Q_{Li} is smaller than P_{Li}, and B_{ij} is negative. η_i should be negative because it is the sensitivity of the system load margin to a change in reactive power demand at bus i. Because when the reactive power demand increases, the load margin should decrease. ς_{ij} is positive because it is the sensitivity of the system load margin to a change in the line thermal limit. If the line thermal limit increases, the load margin should be increased. Therefore equation (47) can be approximated as below:

$$2\mu_{ij}R_{ij} \approx B_{ij}\eta_i + 2c_{ij}\varsigma_{ij} > 0 \tag{48}$$

Therefore, in this case, μij≠0, and all the conic inequality constraints are binding. Consequently, the new model has proven to have zero gap with the original model.

IV. Mixed-Integer Linear Programming

The model in the previous section is a mixed-integer conic programming problem. This section demonstrates that the rotating conic quadratic constraints can be approximated by the polyhedral "ε-relaxed" method where $\varepsilon \in (0,1/2]$ [18]. And the whole OPF problem then is transformed to a mixed-integer linear programming problem which is easier to solve through available software.

The rotating quadratic cone (31) can be expressed as the quadratic cone

$$\sqrt{R_{ij}^2 + T_{ij}^2 + \left(\frac{u_i - u_j}{2}\right)^2} \le \frac{u_i + u_j}{2} \tag{49}$$

The corresponding ε-relaxed approximation of (49) is given by

$$\sqrt{R_{ij}^2 + T_{ij}^2 + \left(\frac{u_i - u_j}{2}\right)^2} \le (1+\varepsilon)\frac{u_i + u_j}{2} \tag{50}$$

This conic inequality constraint can be represented by a set of linear constraints for an arbitrary small value of ε. The polyhedral construction requires expressing (50) by two conic quadratic constraints as shown in (51) and (52).

$$\beta_{ij} \ge \sqrt{R_{ij}^2 + T_{ij}^2} \tag{51}$$

$$\frac{u_i + u_j}{2} \ge \sqrt{\beta_{ij}^2 + \left(\frac{u_i - u_j}{2}\right)^2} \tag{52}$$

Constraints (51) and (52) are the form:

$$\sqrt{x_1^2 + x_2^2} \le x_3 \tag{53}$$

In [18], Ben-Tal and Nemirovski show that this form of inequality constraints can be approximated by a system of linear homogenous equalities and inequalities in terms of x_1, x_2, x_3, and $2(v+1)$ variables φ^k, ψ^k for k =0,…, v. v is a parameter of polyhedral $\varepsilon(v)$-relaxed approximation such that

$$\varepsilon(v) = \frac{1}{\cos\left(\frac{\pi}{2^{v+1}}\right)} - 1 \tag{54}$$

This gives $\varepsilon(v) \approx 1.8\times10^{-8}$ for v =13; the relaxed approximation of (51) and (52) will have $\varepsilon = (1+\varepsilon(v))^2 - 1 \approx 3.6\times10^{-8}$. The system of linear homogeneous equations is given below:

$$\varphi^0 \ge |x_1| \tag{55}$$

$$\psi^0 \ge |x_2| \tag{56}$$

$$\varphi^k = \cos\left(\frac{\pi}{2^{k+1}}\right)\varphi^{k-1} + \sin\left(\frac{\pi}{2^{k+1}}\right)\psi^{k-1}, k = 1,\ldots,v \tag{57}$$

$$\psi^k \ge \left|-\sin\left(\frac{\pi}{2^{k+1}}\right)\varphi^{k-1} + \cos\left(\frac{\pi}{2^{k+1}}\right)\psi^{k-1}\right|, k = 1,\ldots,v \tag{58}$$

$$\varphi^v \le x_3 \tag{59}$$

$$\psi^v \le \tan\left(\frac{\pi}{2^{v+1}}\right)\varphi^v \tag{60}$$

The other constraints of the MILP model for maximizing load margin evaluation are the same with the MICP model in the previous section. They are constraints (33)-(35) and (37)-(44) in Section III. If the complementarity constraints of generators' reactive power limit is relaxed as shown in (22)-(24), the whole model will be converted into a linear programming problem.

V. CASE STUDIES AND RESULTS

In this section, the proposed MICP and MILP based system load margin evaluation is tested on 3 IEEE systems: 14, 30, and 39 bus systems. The results of two relaxations for the complementarity constraints of generators' reactive power limits are also presented. The system data are in [22]. Simulations are run on General Algebraic Modeling System (GAMS) which is a commercial package with capability to solve large scale optimization problems [23]. The original OPF which is a MINLP problem and the proposed conic relaxation model which is a MICP problem is solved by the solver DICOPT with Gurobi as the MIP solver and SNOPF as the NLP solver on NEOS server [24-26]. The MILP model are solved by the solver Gurobi.

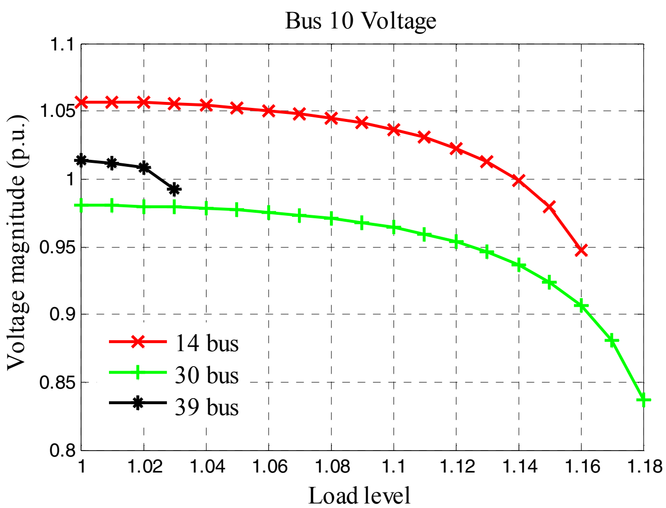

Fig. 1 PV curves of IEEE 14, 30 and 39 bus systems

Fig. 1 shows the PV curves of 3 systems with PV buses voltage fixed which is equivalent with the optimization model of Relaxation 2. The maximum load margins of 3 systems in Fig. 1 are close to the value in Table III, respectively.

The results of system load margin under different methods are in TABLE I, TABLE III and TABLE V. And the corresponding simulation time of each method are in TABLE II, IV and VI.

Table I. Load margin results ignoring complementarity constraints (Relaxation 1)

	NLP	*QCP*	*LP*
14-bus	0.3805	0.3781	0.3781
30-bus	0.5456	0.5448	0.5448
39-bus	0.1451	0.1449	0.1449

Table II. CPU calculation time using different method (relaxation 1)

	NLP	*QCP*	*LP*
14-bus	0.2	0.2	0.3
30-bus	0.2	0.2	0.4
39-bus	0.2	0.2	2.6

The results in TABLE I, III, and IV show that the conic relaxation of the power flow equations can obtain a higher accuracy when the objective is the load margin no matter that the complementarity constraints of reactive power limit of generators are relaxed or not. Therefore, the zero-gap of conic relaxation for power flow equations is obtained as it is discussed in Section III.B. The results of polyhedral approximation of the conic constraints are almost the same with that in the conic relaxation when the polyhedral parameter v is 13 as in the paper.

Table III. Load margin λ results ignoring complementarity constraints (Relaxation 2)

	NLP	*QCP*	*LP*
14-bus	0.1557	0.1553	0.1553
30-bus	0.1624	0.1630	0.1631
39-bus	0.0347	0.0346	0.0346

Table IV. CPU calculation time using different method (Relaxation 2)

	NLP	*QCP*	*LP*
14-bus	0.2	0.1	0.3
30-bus	0.3	0.2	0.6
39-bus	0.4	0.2	4.6

From TABLE I, III, and IV, the results demonstrate that the load margin considering the complementarity constraints of generators' reactive power limit will be much lower than that in the Relaxation 1, but they are larger than that in Relaxation 2. Due to the complexity of the MINLP based ACOPF problem with complementarity constraints, sometimes the global optimal solution is hard to obtain. In this case, the results of load margin of MINLP based ACOPF model are close or same with the results in Relaxation 2 (NLP based ACOPF).

Table V. Load margin λ results considering complementarity constraints

	MINLP	*MIQCP*	*MILP*
14-bus	0.3218	0.3214	0.3214
30-bus	0.1624	0.1630	0.1631
39-bus	0.1168	0.1165	0.1165

Table VI. CPU calculation time using different method

	MINLP	*MIQCP*	*MILP*
14-bus	1.4	1.8	1.1
30-bus	12.6	11.4	6.9
39-bus	18.7	5.1	3.2

TABLE II, IV and VI are the computation time of ACOPF under different method with different relaxations. The calculation time shows that if the complementarity constraints of generators' reactive power limit are relaxed no matter whether the relaxation is Relaxation 1 or 2, the computation time decreases significantly. In some cases, such as IEEE 30 and 118 bus systems, the results of load margin considering the complementarity constraints are same with that in Relaxation 2. While the simulation time is much longer than that in Relaxation 2. Also due to complexity of considering complementarity constraints in a nonlinear programming model, the simulation time increases with the system scale. Therefore, the further simplification of complementarity constraints in the ACOPF model for load margin evaluation is needed in the future work.

VI. CONCLUSIONS

In this paper, a mixed-integer conic programming method to evaluate the system load margin is proposed. The simulation shows that this conic relaxation method can obtain a very high accuracy for load margin evaluation while significantly reducing the calculation time. The polyhedral approximation model of the conic constraints is also proposed, and the results demonstrate that this approximation has achieved a high accuracy same as the conic programming method and decreased the computation time dramtically, especially for the mixed-integer model when the complemetarity constraints of reactive power limits of generators are considered.

Two simplifications and relaxations for the complementarity constraints of reactive power limits of generators in the ACOPF model for load margin evaluation are studied. Relaxation 1 gives a optimistic results while Relaxation 2 leads a conservative results. In some cases, the complexitiy of the problem considering complementarity constraints will lead the solution of MINLP problem a local optima which is the same as in Relaxation 2. Besides, the computation time of mixed-integer models considering the complementarity constraints for large system is still a critical issue which will be the future research work.

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